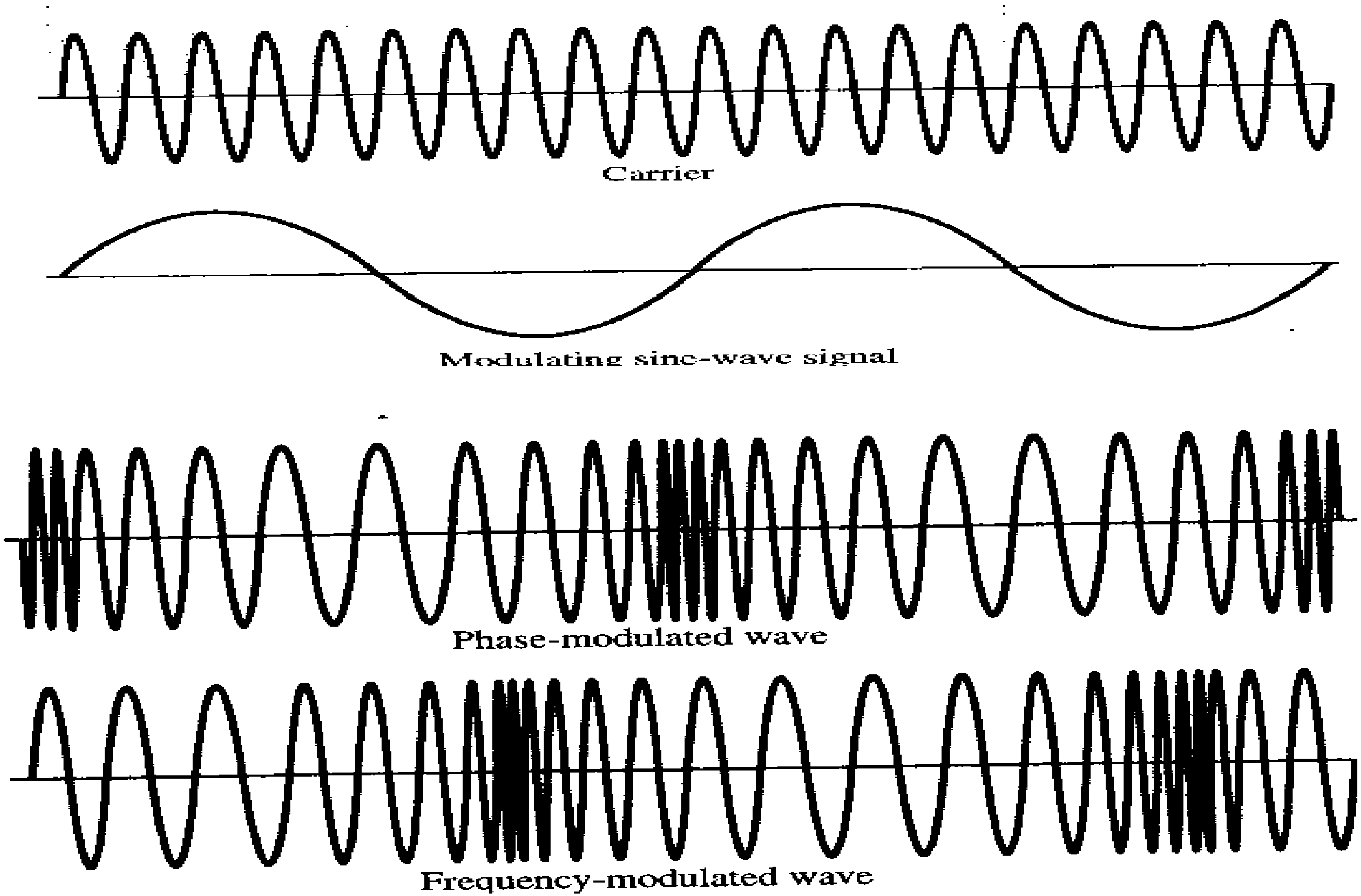
Phase & Frequency Modulation

Phase & Frequency Modulation

- Frequency modulation (FM) and Phase modulation (PM) have much in common and they are usually concluded as angle modulation, i.e., it is impossible to tell them apart without knowledge about the modulation function.
- The carrier amplitude is constant and the phase variation of the signal contains the information.
- A strong motivation for FM and PM is that the amplitude is constant and the transmitter’s power amplifier can work at a constant high amplification level.

Phase & Frequency Modulation



Phase & Frequency Modulation

The angle modulated signal is expressed as:

$$s(t) = a_c \cos(2\pi f_c + \rho(t))$$

When Phase modulation is used, it is the momentary phase of the carrier that carries the information, therefore:

$$\rho(t) = k_\rho m(t)$$

When Frequency modulation is used, it is the momentary frequency of the carrier that carries the information. The momentary frequency ◆(t) is given by the time derivative of the phase:

$$\frac{d\rho(t)}{dt} = \omega(t) = k_f m(t)$$

Phase & Frequency Modulation

The time representation of the angle modulated signals, when the carrier is $\boldsymbol{a_c cos(2\pi f_c t)}$ and the message signal is ***m(t)*** is given by:

$$s(t) = a_c \cos(2\pi f_c t + k_p m(t)) \qquad \text{PM}$$

$$s(t) = a_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau\right) \qquad \text{FM}$$

Where $\boldsymbol{k_f}$ and $\boldsymbol{k_p}$ represent the ***deviation constants*** of PM and FM respectively. These deviation constants are ***implementation dependent,*** i.e., they depend on the modulator circuit.

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The ***frequency-domain representation*** of angle modulated signals is, in general, very complex due to the nonlinearity of these modulation schemes. Therefore we only consider the case where the message signal *m(t)* is a sinusoidal signal.

We therefore assume ***m(t)=*** $\boldsymbol{a_m}$***cos(2□*** $\boldsymbol{f_m}$***t)*** for PM and ***m(t)=-*** $\boldsymbol{a_m}$***sim(2□*** $\boldsymbol{f_m}$***t)*** for FM. Then the modulated signal is on the form:

$$s(t) = a_c \cos(2\pi f_c t + \mu_\rho \cos(2\pi f_m t)) \qquad \text{PM}$$

$$s(t) = a_c \cos(2\pi f_c t + \mu_f \cos(2\pi f_m t)) \qquad \text{FM}$$

Where ***Ο***$_{\boldsymbol{p}}$***, Ο***$_{\boldsymbol{f}}$ is the ***modulation index*** of PM and FM.

$$\mu_\rho = k_\rho a_m \qquad\qquad \mu_f = \frac{k_f a_m}{f_m}$$

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In general, for a non sinusoidal *m(t)*, the modulation index, ❍, is defined as:

$$\mu_\rho = k_\rho \max\left|m(t)\right|$$

$$\mu_f = \frac{k_f \max\left|m(t)\right|}{W}$$

where *W* is the bandwidth of the message signal *m(t)*.

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However, in the case of a sinusoidal message signal, the modulated signal can be represented as:

$$s(t) = \sum_{n=-\infty}^{\infty} a_c J_n(\mu)\cos(2\pi(f_c + nf_m)t)$$

Where J_n(❍) is the ***Bessel function*** of the first kind and of order *n* and ❍ is either ❍$_p$ or ❍$_f$ depending on whether we are dealing with PM or FM.

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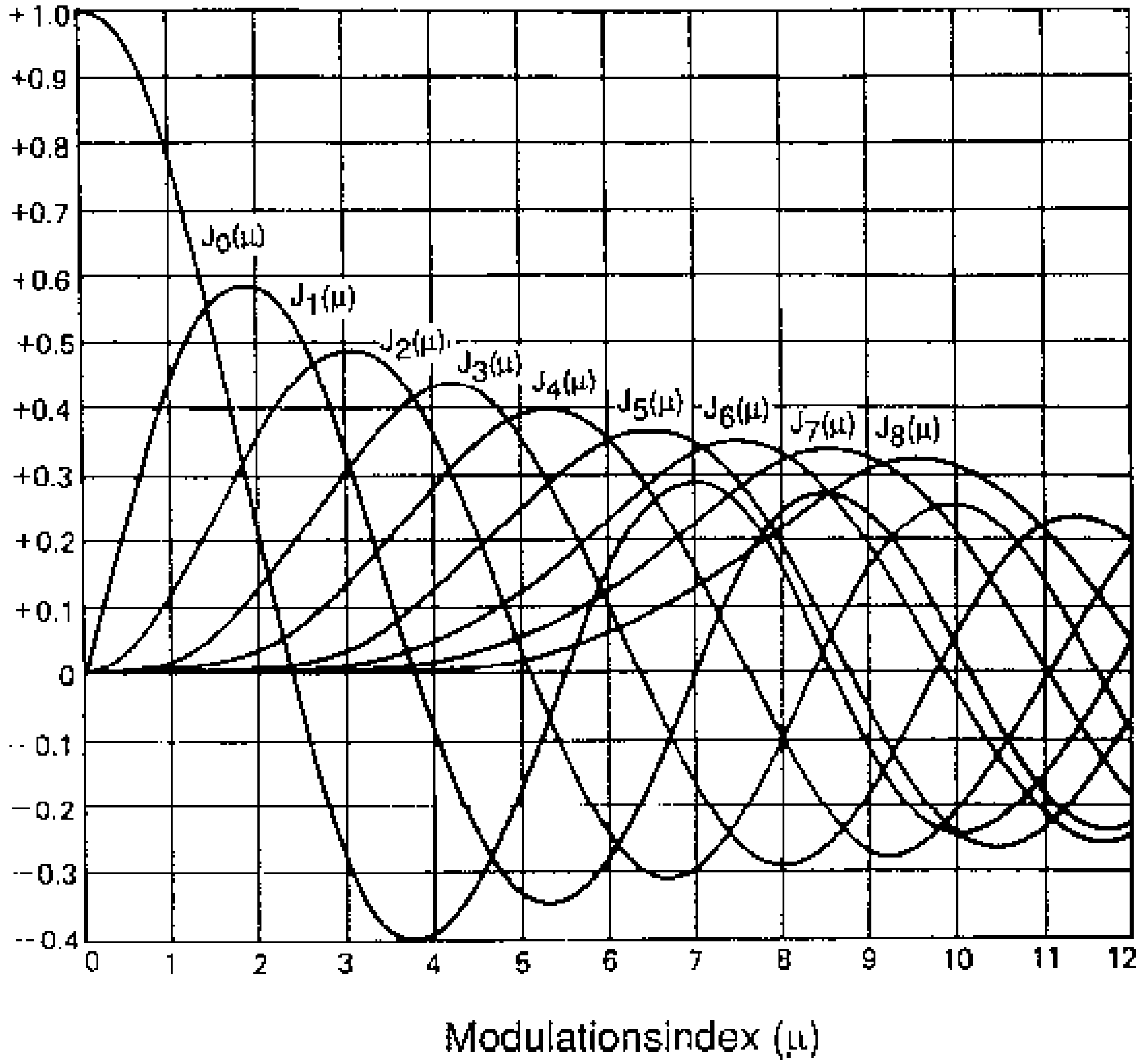

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In the frequency domain we have:

$$S(f) = \sum_{n=-\infty}^{\infty} \left(\frac{a_c J_n(\mu)}{2} \delta(f - (f_c + nf_m)) + \frac{a_c J_n(\mu)}{2} \delta(f + (f_c + nf_m)) \right)$$

Forget it !!

Obviously, the bandwidth of the modulated signal is not finite.

However, we can define the ***effective bandwidth*** of the signal as the bandwidth containing 98% to 99% of the modulated signal power. The bandwidth is then given according to ***Carson’s rule*** as:

$$B = 2(\mu + 1)W$$

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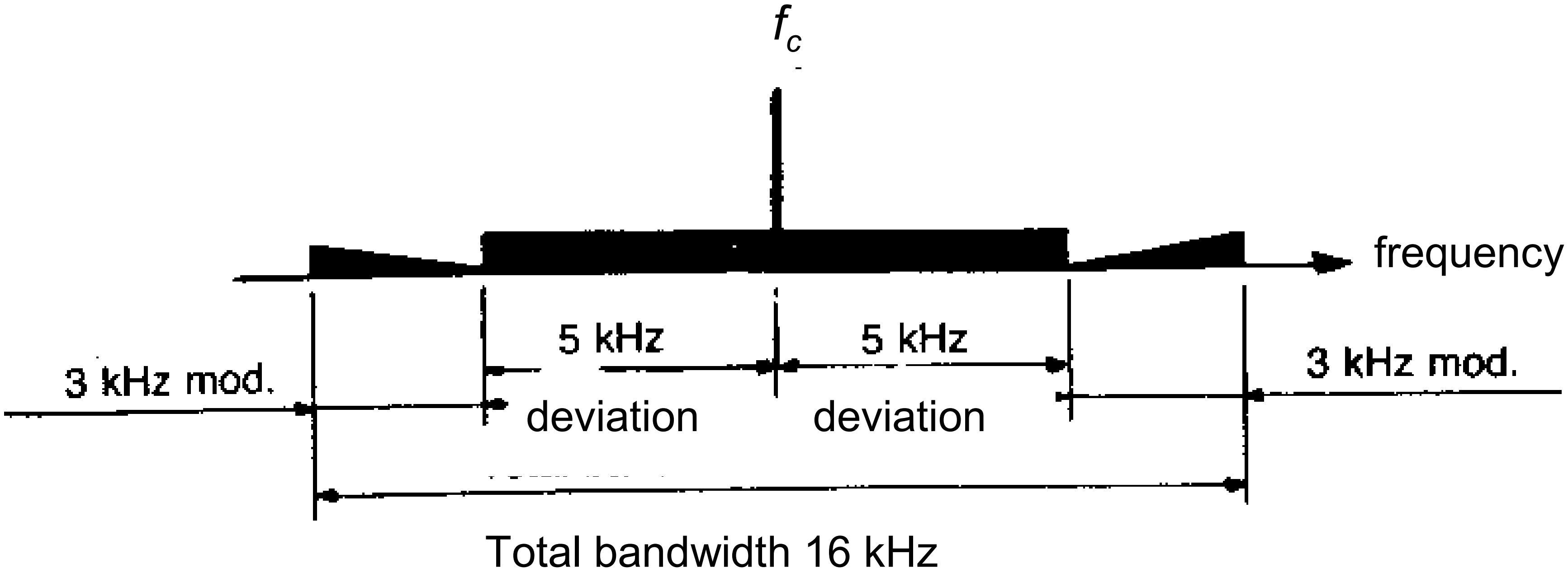
A graphical example:

Message bandwidth 3 kHz

Deviation (swing) 5 kHz



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Since, the modulated signal is sinusoidal with varying instantaneous frequency and constant amplitude, its power, P_u, is constant and ***does not*** depend on the message signal. The power content for both PM and FM is:

$$P_u = \frac{a_c^2}{2}$$

However the SNR for angle modulated signals is given by:

$$\left(\frac{S}{N}\right)_p = \frac{P_m \mu_p^2}{\left(\max|m(t)|\right)^2} \frac{P_r}{N_0 W} \quad \text{PM}$$

$$\left(\frac{S}{N}\right)_f = 3\frac{P_m \mu_f^2}{\left(\max|m(t)|\right)^2} \frac{P_r}{N_0 W} \quad \text{FM}$$

Since ***max|m(t)|*** denotes the maximum magnitude of the message signal, we can interpret ***P_m/max|m(t)|*** *as the normalized message signal.*