Digital modulation

Digital modulation

- In a digital communication system, a finite number of electrical waveforms (symbols), are transmitted over the communication channel.
- Each symbol can represent one or more bits.
- The modulator in a digital communication system map the digital information into analogue waveforms that match the channel.
- The job of the receiver is to estimate which symbol that was originally sent by the transmitter, i.e., after noise and interference has been added.
- It is not important what size (amplitude) or what shape the received signal has, as long as the receiver can clearly distinguish one symbol from another.

Digital modulation

$$s(t) = A\sin(2\pi f_c t + \rho)$$

By letting the information control the amplitude, frequency or phase we get the three fundamental cases of digital modulation:

- Amplitude shift keying (ASK)
- Frequency shift keying (FSK)
- Phase shift keying (PSK)

We will start by investigating how the digital signal signal is effected in the baseband.

Baseband digital transmission

In a binary communication system, binary data consisting of a sequence of ’0’s and ’1’s are transmitted by means of two signal waveforms $s_0(t)$ and $s_1(t)$. Further, suppose that the data rate is specified as R bits per second. Then each bit is mapped into a corresponding signal waveform according to the rule:

$$0 \rightarrow \mathrm{S}_0(\mathrm{t}) \,, \quad 0 \le t \le T_b$$

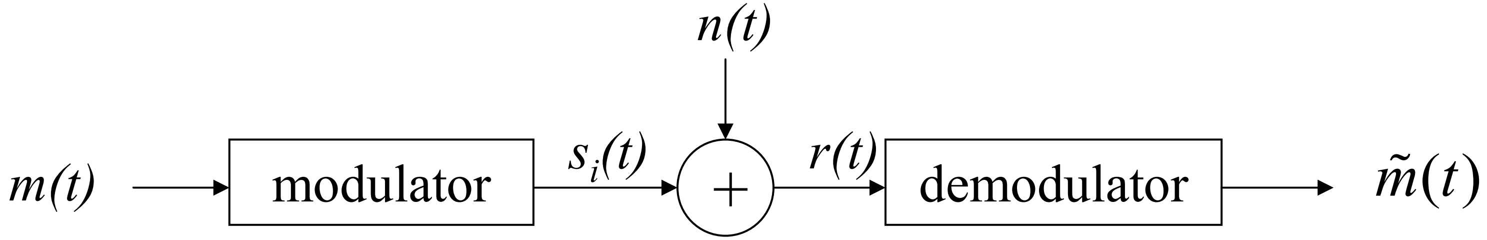
$$1 \rightarrow \mathrm{S}_1(\mathrm{t}) \,, \quad 0 \le t \le T_b$$

Where T_b=1/R is defined as the bit time interval. We further assume that the data bits ’0’ and ’1’ is equally probable and mutually statistically independent.

Baseband digital transmission

We assume that the channel through which the signal is transmitted is corrupted by white Gaussian noise, denoted *n(t)*, with a power spectrum of $N_0/2$ [watt/hertz].

Such a channel is called additive white Gaussian noise (AWGN).



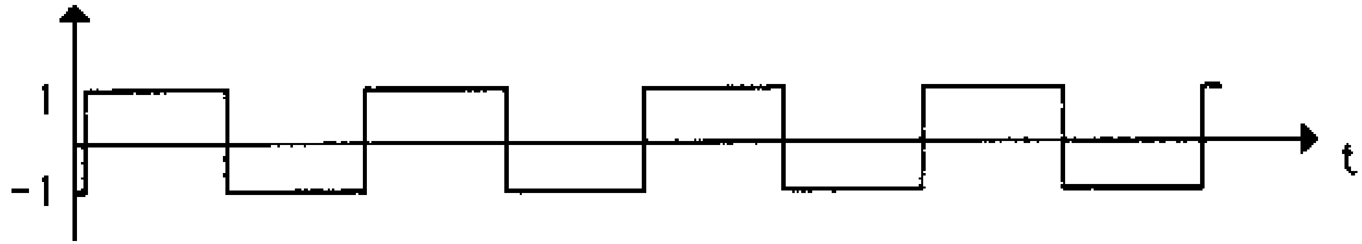
Consequently the received signal wave form is expressed as:

$$r(t) = s_i(t) + n(t) \qquad i = 0, 1 \quad 0 \le t \le T_b$$

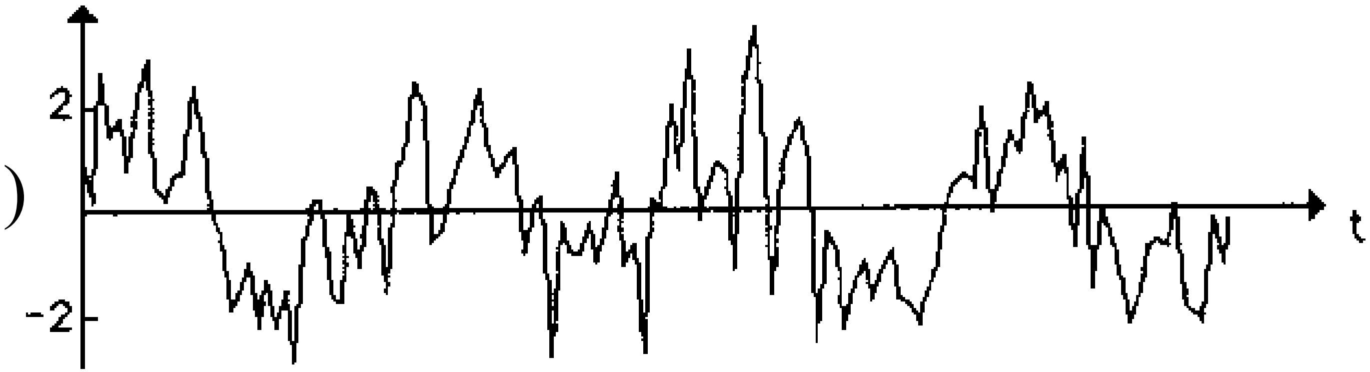
Baseband digital transmission

Considering a pulse train where ’0’ is –1 volt and ’1’ is +1volt.

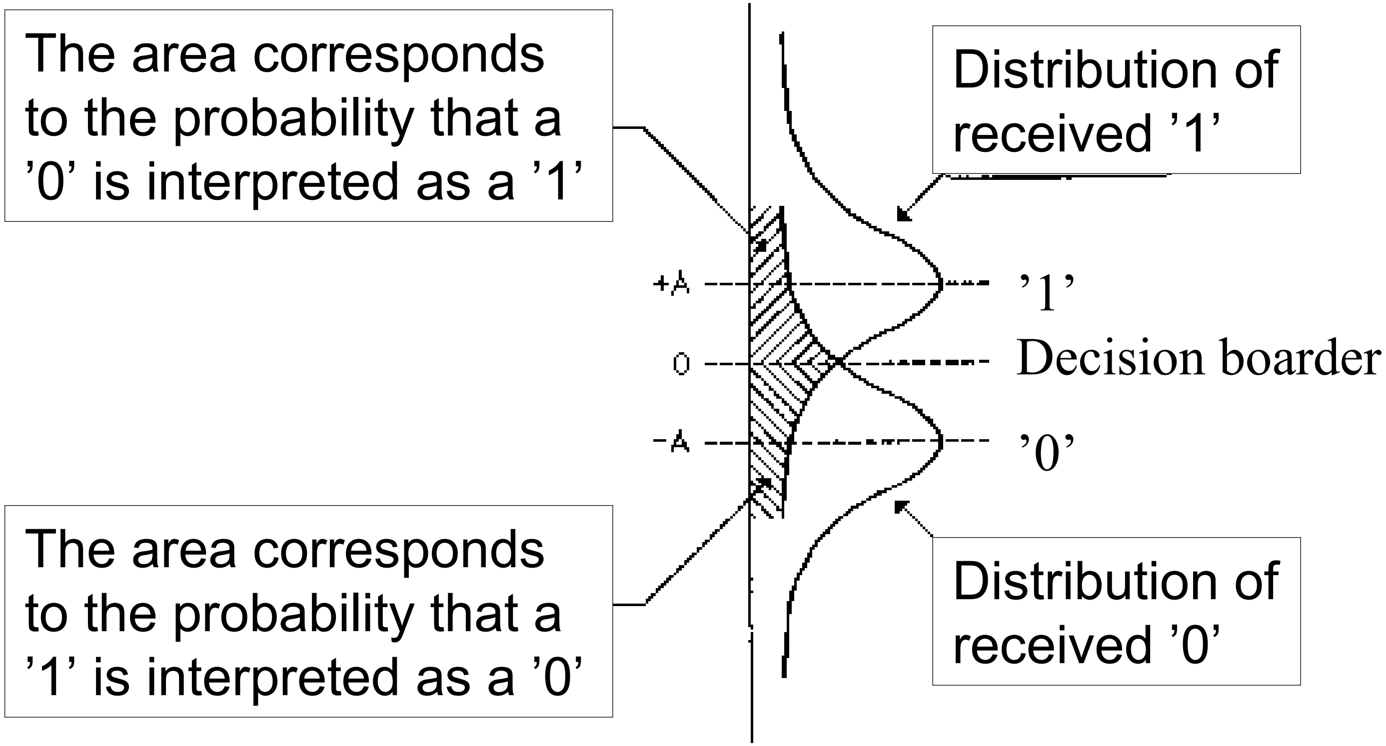
$s_i(t)$



$$r(t) = s_i(t) + n(t)$$



Baseband digital transmission



Baseband digital transmission

The probability that a ‘0’ is detected as a ‘1’, is possible to determine by integrating the probability density function *p(r)* for a ‘0’ from the decision boarder until +∞.

We define this probability as P(1 was detected | 0 was transmitted), i.e., suppose that a ‘0’ is transmitted what is the probability that it will be detected as a ‘1’. This is called the conditional probability of ‘0’ given the occurrence of ‘1’.

$$P(1|0) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^{\infty} e^{\frac{-(r+A)^2}{2\sigma^2}} dx$$

The Gaussian noise has a average of 0 and a deviation of ♦.

Baseband digital transmission

In the same way one can calculate the opposite, the probability that '1' is detected as '0'.

$$P(0|1) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{0} e^{\frac{-(r-A)^2}{2\sigma^2}} dx$$

The total ***error probability*** is then:

$$P_e = P(1)P(0|1) + P(0)P(1|0)$$

where *P(0)* and *P(1)* is the probability for '0' and '1' respectively.

Baseband digital transmission

If we assume that the ‘1’ and ‘0’ is equally probable:

$$P(1) = P(0) = \frac{1}{2}$$

Further, the symmetry of P(0|1) and P(1|0) gives that we can express them as:

$$P(0\,|\,1) = P(1\,|\,0) = \frac{1}{\sqrt{2\pi}} \int_{A/\sigma}^{\infty} e^{\frac{-x^2}{2}} dy = Q\left(\frac{A}{\sigma}\right)$$

Once more we can say skip the integral!!!

The integral is actually not solvable and must be calculated numerically!

However the integral is given by the so called Q-function which is a tabulation of the solution of the integral.

Baseband digital transmission

When we exchange the integral by the tabulated Q value we can calculate the total error probability:

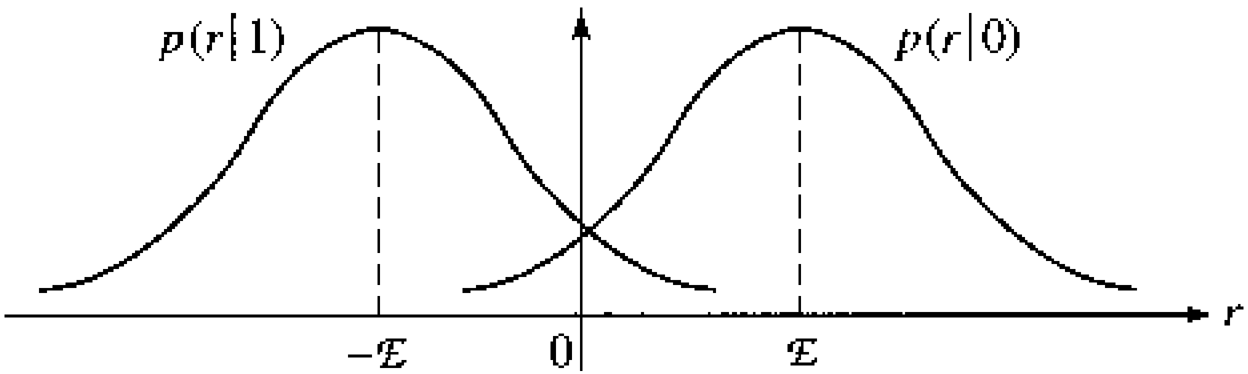
$$P_e = \frac{P(0|1)}{2} + \frac{P(1|0)}{2} = \frac{Q\left(\frac{A}{\sigma}\right)}{2} + \frac{Q\left(\frac{A}{\sigma}\right)}{2} = Q\left(\frac{A}{\sigma}\right)$$

An identification of the parameters shows that the input value to the Q-function is actual the E_b/N_0.

$$P_e = Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Baseband digital transmission

This error probability only holds for a certain ***signal constellation***. In this case the signals s_0 and s_1 corresponds to the signal *–s(t)* and *s(t),* each having the energy *E*, the two signal points then falls on the real line at √ E and –√E.

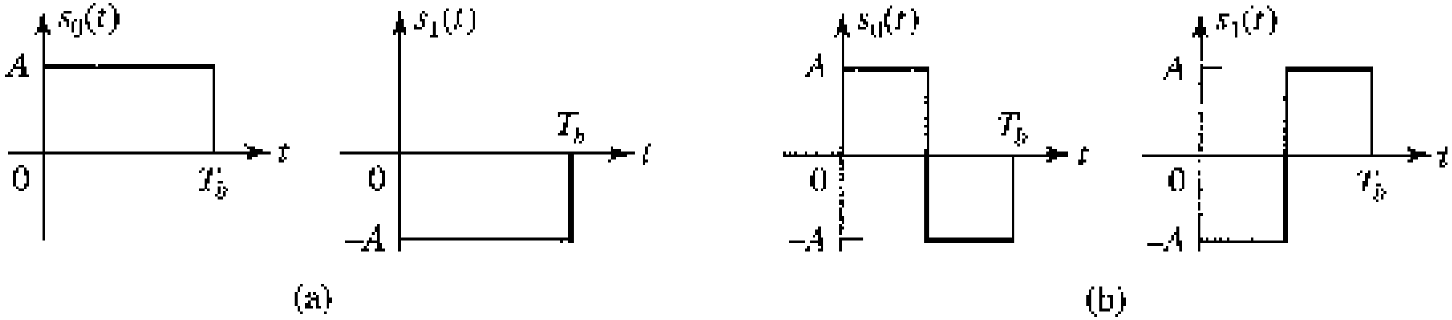


This signal constellation is referred to as ***antipodal***.

We actually characterize the signals geometrically by putting them in a signal space. The one dimensional signal space follows from the fact that only one signal wave form suffice to represent the antipodal signals.

Baseband digital transmission

This is examples of antipodal signals

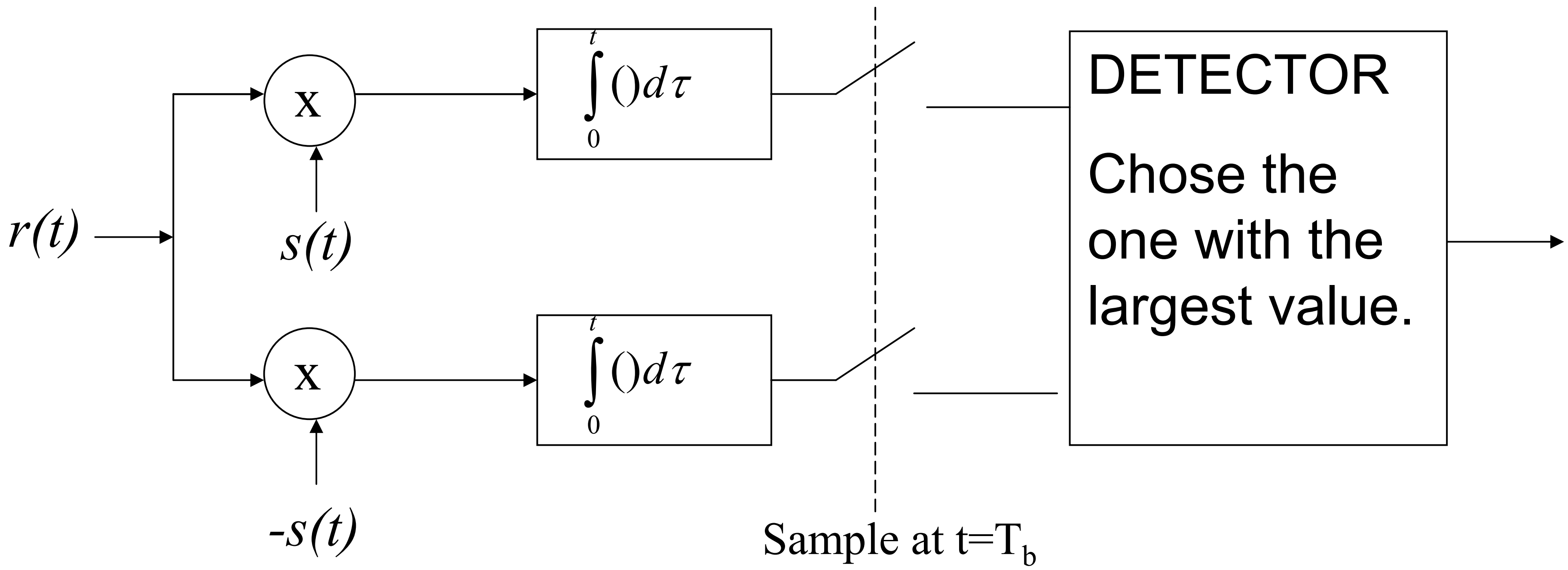


So we can actually express the received signal as:

$r(t) = \pm\, s(t) + n$

Baseband digital transmission

The task of the receiver is to determine whether a '0' or '1' was transmitted after observing the received signal r(t). One receiver that provides the optimal estimate, assuming an AWGN channel, of the received signal *r(t)* is the so called signal correlator.



Baseband digital transmission

The signal correlator cross-correlates the received signal with the two possible signals *s(t)* and *–s(t)*, for antipodal signals. That is, the signal correlator computes the two outputs:

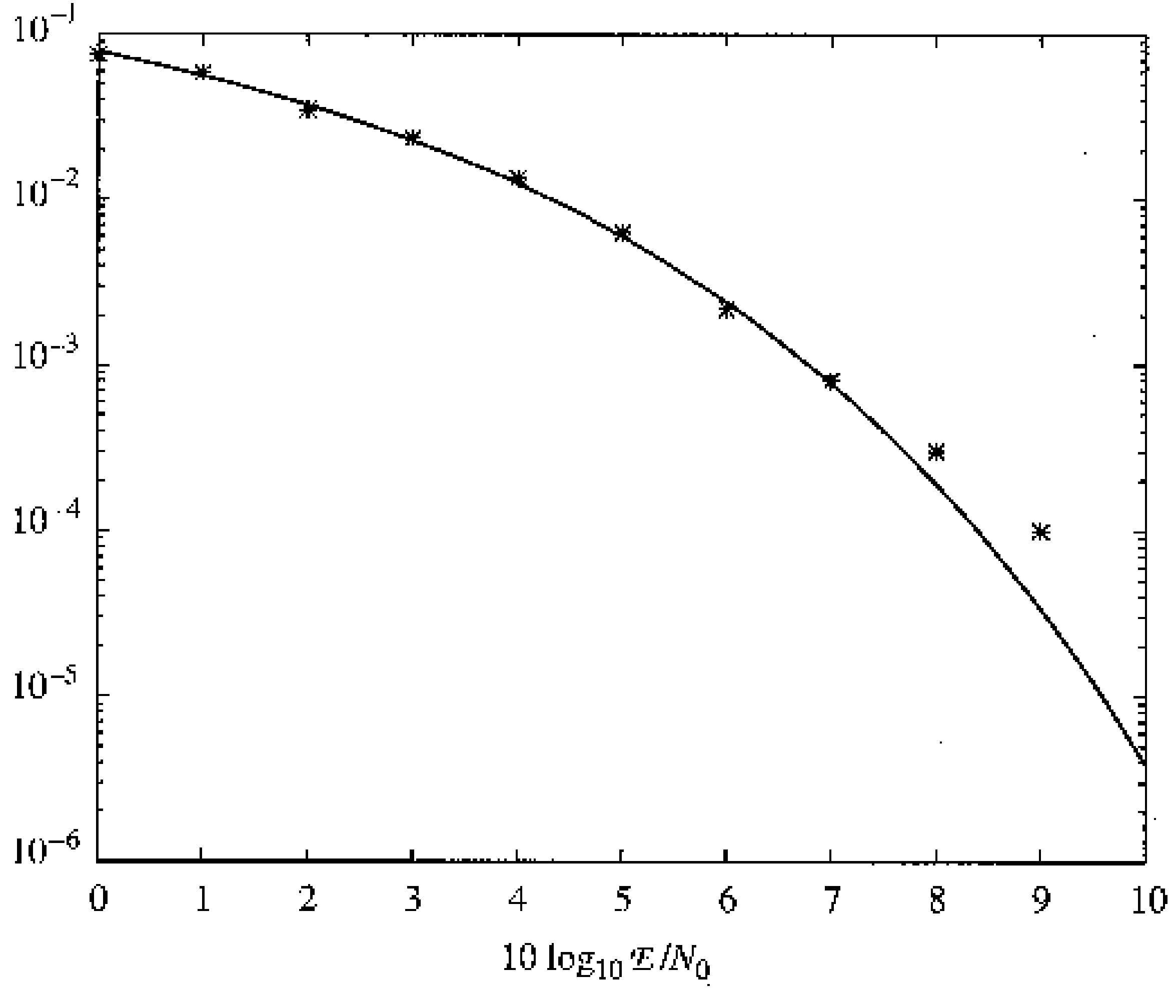
$$r_0(t) = \int_0^t r(\tau)s(\tau)d\tau$$

$$r_1(t) = \int_0^t r(\tau)(-s(\tau))d\tau$$

In the interval $0<t<T_b$, samples the two outputs at T_b and feeds the two outputs to the detector. The detector chooses the one with the highest value.

Baseband digital transmission

Error probability P_e versus E_b/N_0 for antipodal signaling



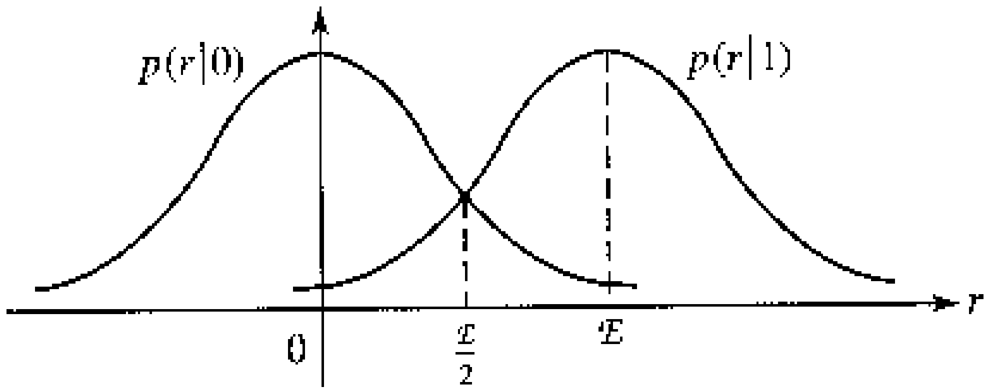
Baseband digital transmission

If we now consider another signal scheme, ***on-off keying***. To transmit a ‘0’ no signal is transmitted in the time interval T_b, to transmit a ‘1’ a signal waveform s(t) is transmitted. Consequently the received signal form may be represented as:

$$r(t) \begin{cases} n(t) & \text{if a ‘0’ is transmitted} \\ s(t) + n(t) & \text{if a ‘1’ is transmitted} \end{cases}$$

As in the case of antipodal signals, the optimum receiver consists of a correlator, whose output is sampled at $t=T_b$, and followed by a detector that compares the sampled output to the threshold, denoted as ♋. In this case if $r >$ ♋, a ‘1’ is declared to have been transmitted; otherwise a ‘0’ is declared to have been transmitted.

Baseband digital transmission

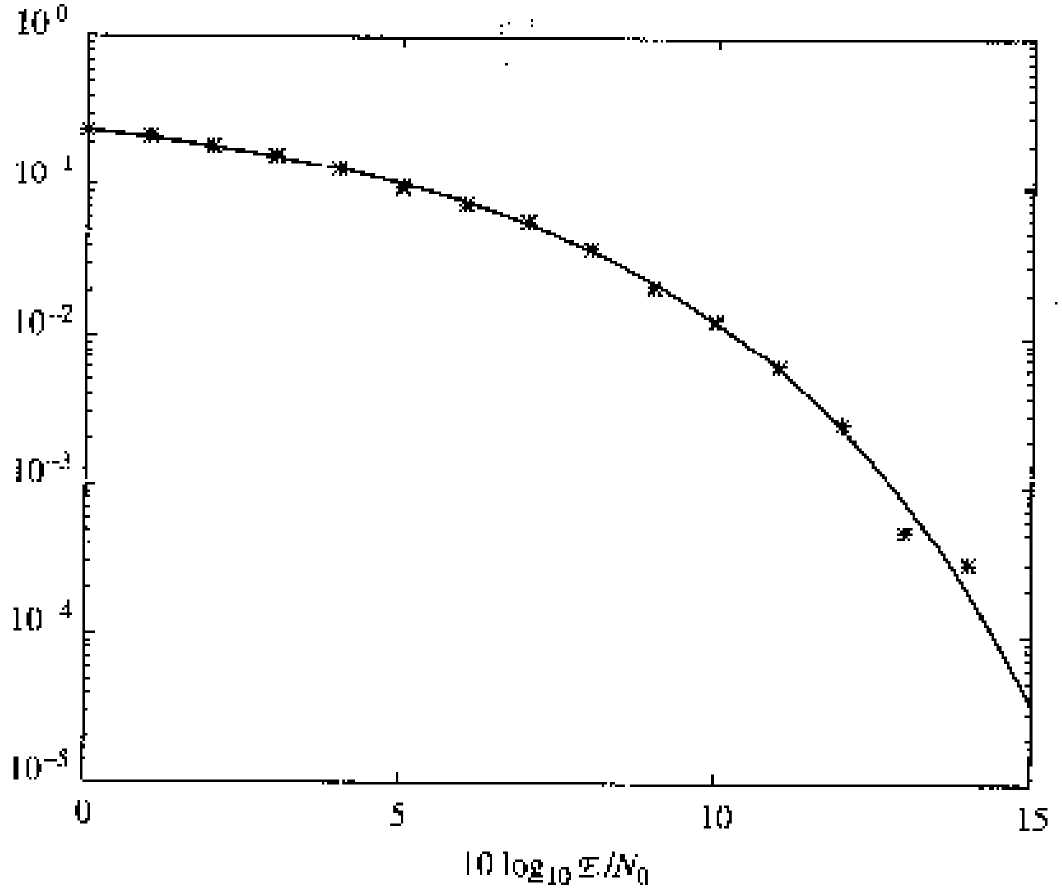


On-off signals are also one dimensional. Hence the two signal points fall on the real line at 0 and *E*.

$$P_e = Q\left(\sqrt{\frac{E}{2N_0}}\right)$$

Baseband digital transmission

Error probability P_e versus E_b/N_0 for on-off signaling

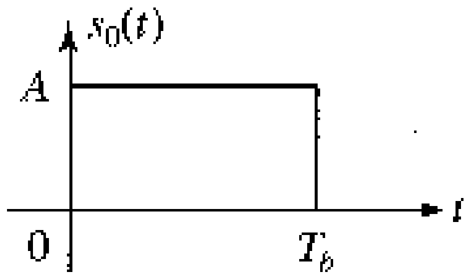


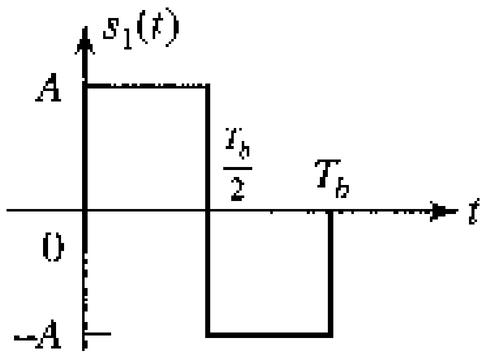
Baseband digital transmission

The third and last signaling scheme for binary signals is based on ***orthogonal signals***.

$$r(t) \begin{cases} s_0(t) + n(t) & \text{if a '0' is transmitted} \\ s_1(t) + n(t) & \text{if a '1' is transmitted} \end{cases}$$

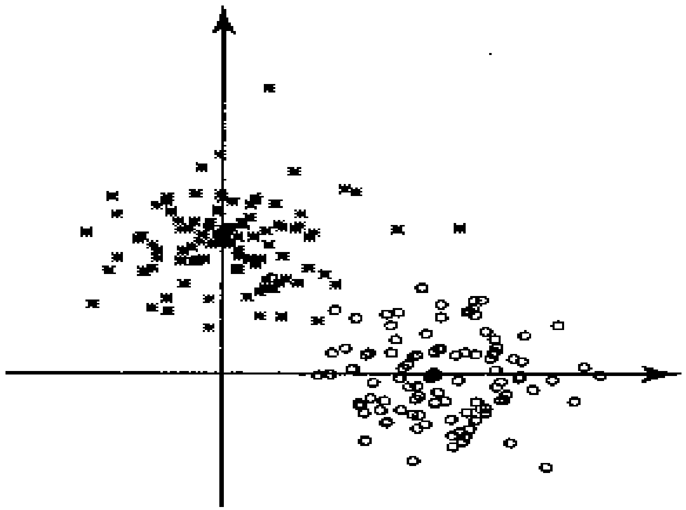
Examples of orthogonal signals are:

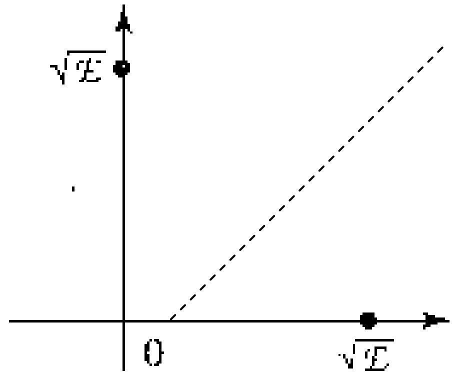




Baseband digital transmission

Binary orthogonal signals require a two-dimensional geometric representation, since there are two linear independent functions *$s_0(t)$* and *$s_1(t)$* that constitutes the two signal waveforms.





$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Baseband digital transmission

Error probability P_e versus E_b/N_0 for orthogonal signaling