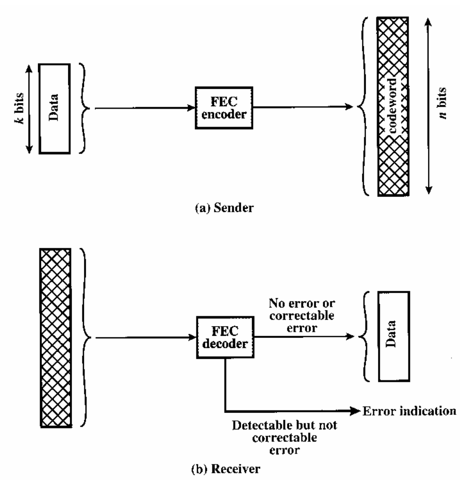
Correction codes

Correction codes

Correction of errors using an error detection code requires that block of data is re-transmitted, using ARQ schemes. For wireless applications this approach is inadequate for different reasons:

- The bit error rate on a wireless link can be quite high, which would result in a large number of retransmissions.
- In some cases, especially satellite links, the propagation delay is very long compared to the transmission time of a single frame. The result is a very inefficient system.
- Some systems do not have any bidirectional channel for control messages, i.e., broadcast systems.

Correction codes



On the transmitter side, each *k* block of data is mapped into a *n* bit block called a **code word**, using a forward error correction encoder.

The code word is transmitted over the channel where it is subject to noise and interference, which may produce bit errors.

At the receiver the received bit string is passed through the FEC decoder.

Correction codes

The FEC decoder decodes the received code words, with one of four possible outcomes:

- If there are no bit errors, the input to the FEC decoder is identical to the original code word, and the decoder produces the original data block as output.
- For certain error patterns, it is possible for the decoder to detect and correct those errors. Thus, even though the incoming data block differs from the transmitted code word, the FEC decoder is able to map this block into the original data block.

Correction codes

- For certain error patterns, the decoder can detect but not correct the errors. In this case, the decoder simply reports an uncorrectable error.
- For certain, typically rare error patterns, the decoder does not detect that any errors have occurred and maps the incoming n-bit data block into a k-bit block that differs from the original.

Correction codes

- Assume that the information stream is divided into blocks with the length k.
- The number of symbols in the channel coded message is expanded with control symbols.
- This increases the number of symbols in a block to n symbols.
- This is referred to as a (n,k) code.
- With an (n,k) block code there are 2^k **valid code words** out of a total of 2^n **possible code words**.

Double parity check

Double (15,9) parity check, both rows and columns have parity bits.

m_1	m_2	m_3	c_1
m_4	m_5	m_6	c_2
m_7	m_8	m_9	c_3
c_4	c_5	c_6	

$C_1 = m_1 \oplus m_2 \oplus m_3$

$C_2 = m_4 \oplus m_5 \oplus m_6$

$C_3 = m_7 \oplus m_8 \oplus m_9$

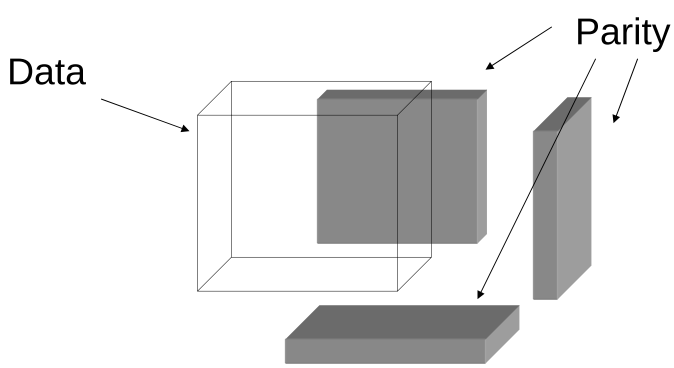
$C_4 = m_1 \oplus m_4 \oplus \mathrm{m}_7$

$C_5 = m_2 \oplus m_5 \oplus m_8$

$C_6 = m_3 \oplus m_6 \oplus m_9$

Multi-dimensional parity check

3-dimensional parity check
(54, 27)

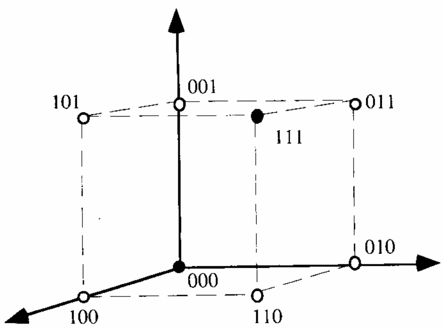


Repetition code

The idea of channel coding is that the number of combinations symbols has been limited in such a way, that the minimum Euclidian distance between two symbols are maximized.

A simple example is a (3,1) repetition code.

As code words ’000’ and ’111’ are chosen.



(3,1) kod

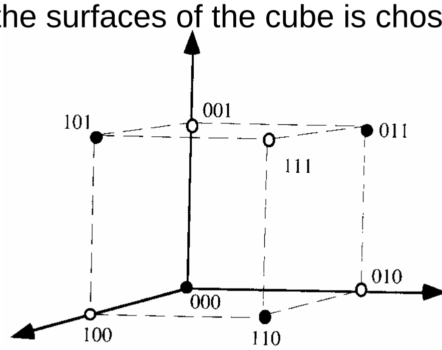
- The code words spans a cube.
- These two code words have maximum distance out of all possible code words.

Trivial codes and definitions

If we continue with the previous example, (3,1) repetition code, and assumes that we want two information bits to be coded with three bits, a (3,2) code.

The cube diagonal can obviously not be chosen.

Instead the diagonal of the surfaces of the cube is chosen as separation distance.



(3,2) kod

Trivial codes and definitions

- In these previous examples with code words constructed out of 3 bits the problem of finding optimal placement of the code words is trivial.
- However in order to get better performance in a communication system the codes is made longer, which means that the number of dimensions that the all possible code words spans increase.
- If we also add the demand that the coder and decoder should have low complexity will the problem of finding good codes become non trivial.

Trivial codes and definitions

The ratio of redundant bits to data bits :

$$\frac{n-k}{k}$$

is called the **redundancy** of the code.

The ratio of data bits to total bits is called **code rate**:

$$R_c = \frac{k}{n}$$

The code rate is a measure of how much **additional bandwidth** that is required to carry data at the same data rate as without the code.

For example, a code rate of ½ requires the double bandwidth of an uncoded system to maintain the same data rate.

Trivial codes and definitions

Hamming distance, d($\mathbf{v}_1$,$\mathbf{v}_2$), between two *n*-bit binary sequences $\mathbf{v}_1$ and $\mathbf{v}_2$ is the number of bits in which $\mathbf{v}_1$ and $\mathbf{v}_2$ disagree.

Example assume

$\mathbf{v}_1$=011011 and $\mathbf{v}_2$=110001

then d($\mathbf{v}_1$,$\mathbf{v}_2$) = 3

For a code consisting of the code words $\mathbf{W}_1$,$\mathbf{W}_2$….$\mathbf{W}_s$, where $s = 2^n$, the minimum distance, d_{min}, of the code is defined as:

$$d_{\min} = \min_{i \neq j}\left[d(\mathbf{W}_i, \mathbf{W}_j) \right]$$

Trivial codes and definitions

For a given positive integer *t*, if a code satisfies $d_{min} \geq 2t + 1$, then the code can correct all bit errors up to *t* and including errors of *t* bits.

If $d_{min} \geq 2t$, then all errors $\leq t - 1$ bits can be corrected and all errors of *t* bits can be detected but not corrected.

Another way of putting the relation between d_{min} and *t* is to say that the maximum numbers of guaranteed correctable errors per code word satisfies:

$$t = \left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor$$

The numbers of errors, *t*, that can be detected satisfies:

$$t = d_{\min} - 1$$

Trivial codes and definitions

The design of a block code involves a number of considerations:

- For given values of ***n*** and ***k***, we would like the largest possible value of d_{min}.
- The code should be relatively easy to encode and decode, requiring minimal memory and processing time.
- We would like the number of extra bits, (n-k), to be small to reduce bandwidth.
- We would like thw number of extra bits, (n-k), to be large to reduce error rate.

Clearly the last two objectives are in conflict and a trade off must be made.

Trivial codes and definitions