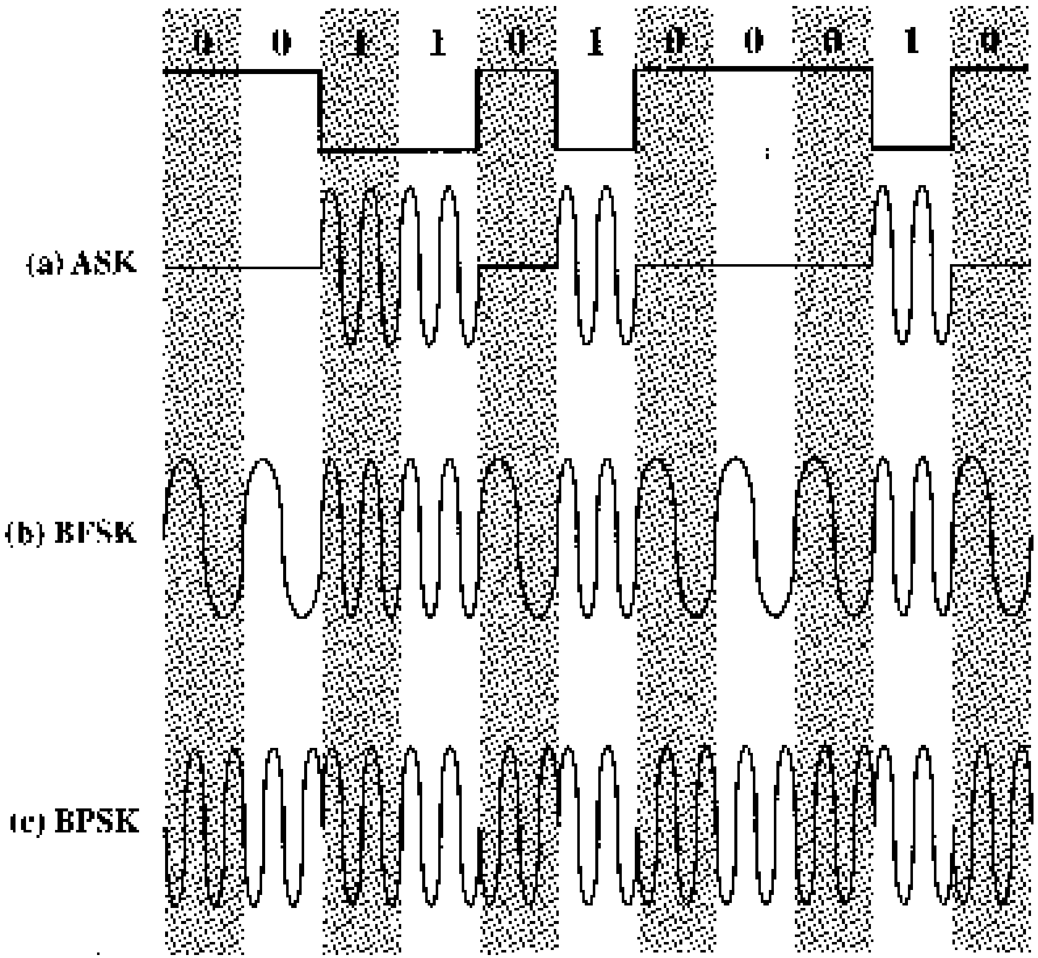
Digital transmission via carrier modulation

Digital modulation



Amplitude Shift Keying

In amplitude shift keying two binary values are represented by two different amplitudes of the carrier frequency.

Commonly one of the amplitudes is zero and one is represented by the presence of the carrier, on-off keying.

$$s(t) \begin{cases} A\cos(2\pi f_c t) & \text{'1'} \\ 0 & \text{'0'} \end{cases}$$

We know that this kind of modulation scheme has an error probability of:

$$P_e = Q\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Frequency Shift Keying

In a general frequency shift keying (FSK) system the signals are defined by:

$$s(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_i t) \qquad i = 1,2 \ldots\ldots M$$

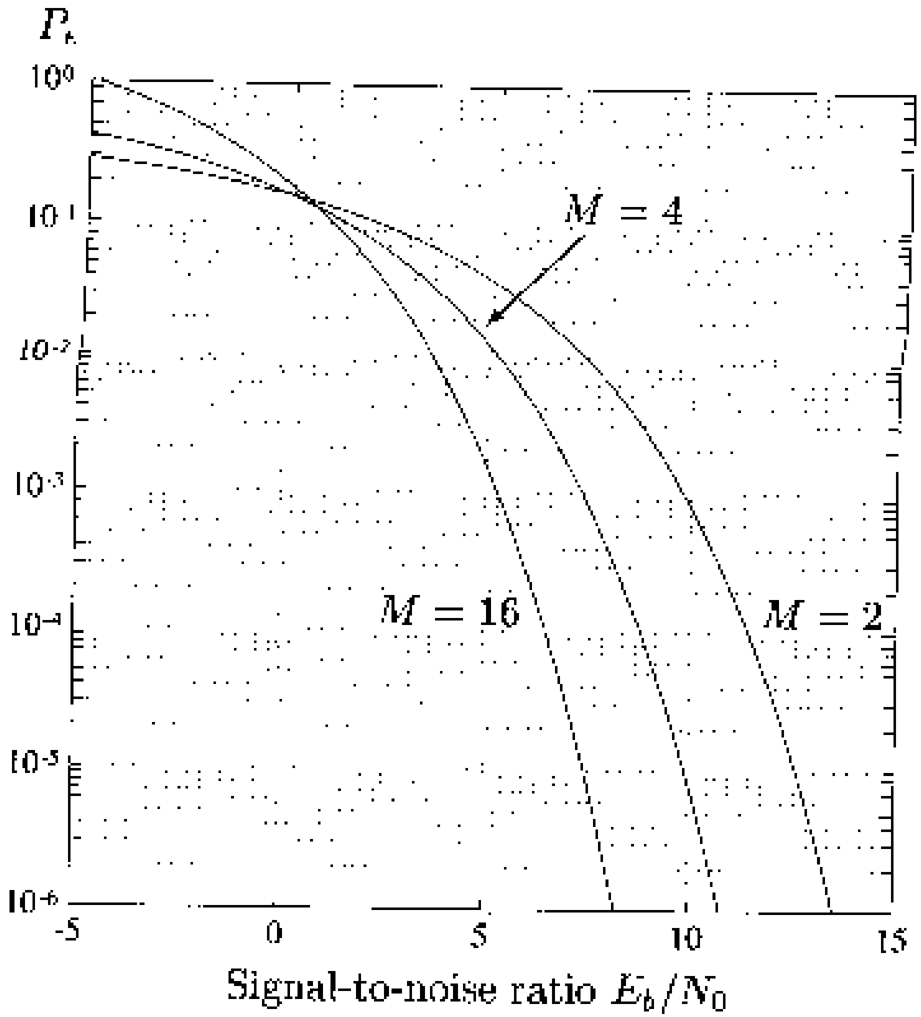
Where the distance between the frequencies is $1/2T_s$ Hz in order to obtain orthogonality.

The number of frequencies is often chosen to be $M=2^K$. Then, every transmitted symbol corresponds to K data bits.

The ***symbol rate*** ,R_s , and the ***bit rate*** ,R_b ,are related by $R_s=R_b/K$. The bit error probability for the general M-ary FSK with coherent detection is given by:

$$P_e = \frac{M}{2} Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

Frequency Shift Keying



Frequency Shift Keying

The power spectrum of M-ary FSK is complicated to derive, for the case when M signal frequencies are separated by $1/2T_s$ we can approximate the bandwidth to:

$$B = \frac{R_b(M+3)}{2\log_2 M}$$

Binary Phase Shift Keying

For a binary phase shift keying (BPSK) system, the phase of a constant envelop is switched between two phase values corresponding to binary symbols ‘0’ and ’1’ respectively. The transmitted BPSK signal $s_0(t)$ and $s_1(t)$ are given by:

$$s_0(t) = \sqrt{\frac{2E_b}{T_b}\cos(2\pi f_c t)}$$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

The bit error probability of coherently detected BPSK is:

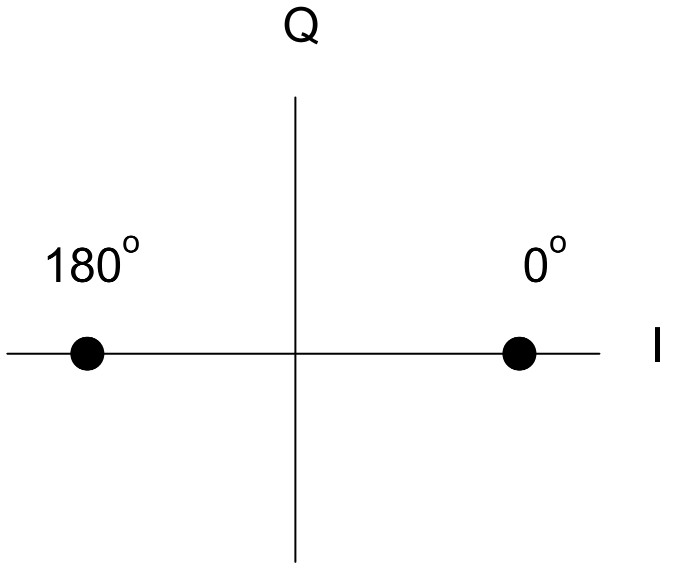
$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

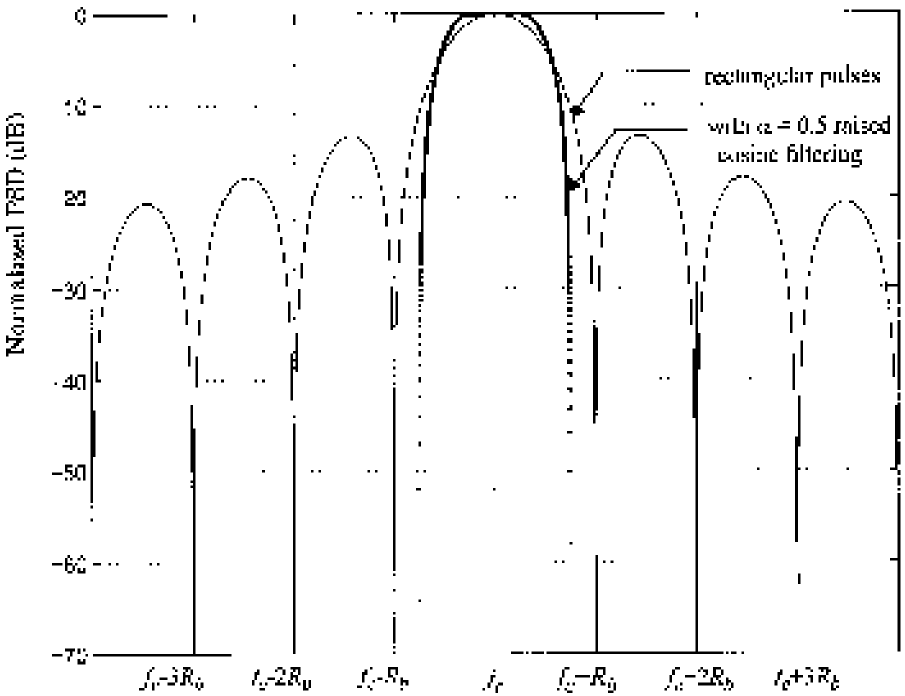
BPSK is antipodal signaling.

Binary Phase Shift Keying

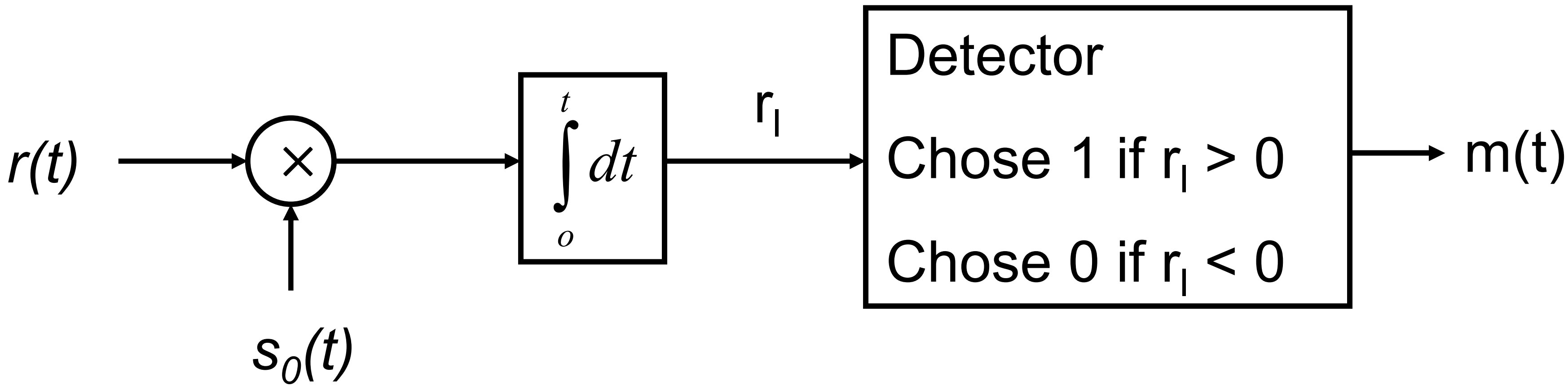
The power spectral density of the baseband signal can be expressed as:

$$\theta(f) = 2E_b\left(\frac{\sin(2\pi fT_b)}{(\pi fT_b)}\right)^2$$

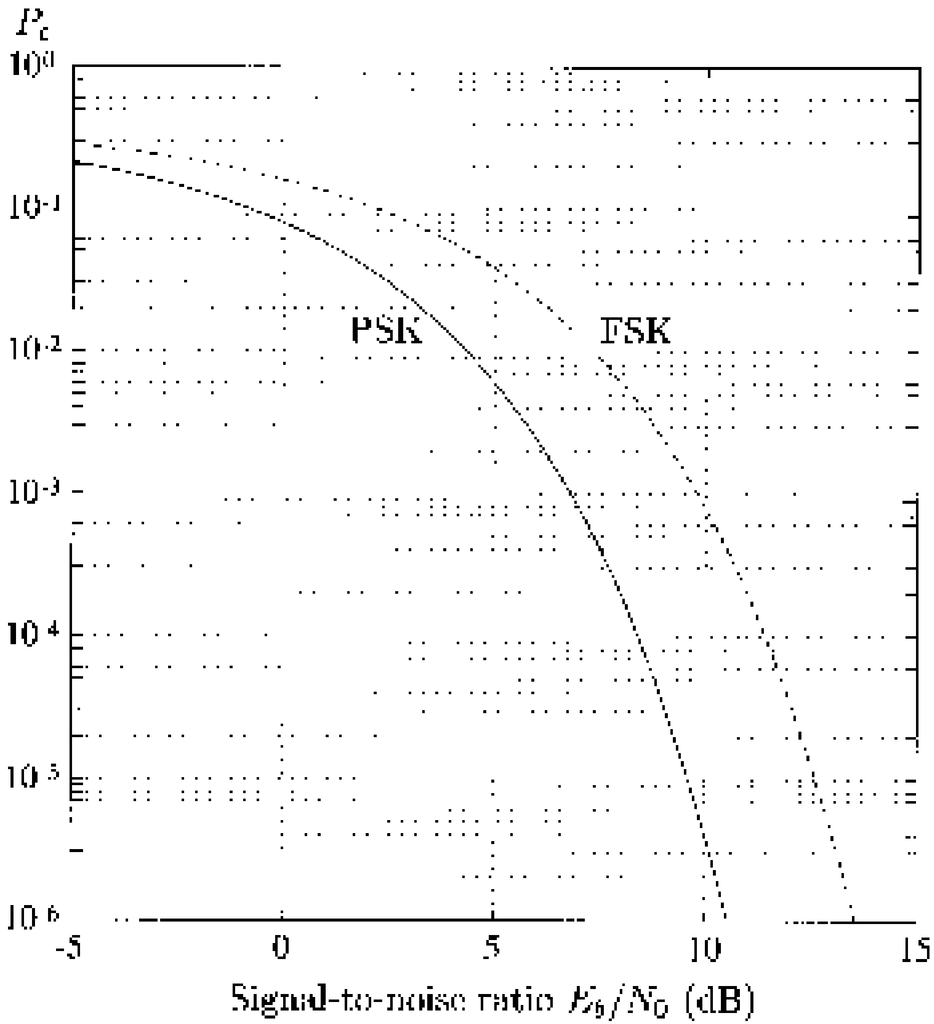




Binary Phase Shift Keying



BPSK – BFSK



P_e
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
PSK
FSK
-5
0
5
10
15
Signal-to-noise ratio E_b/N_0 (dB)

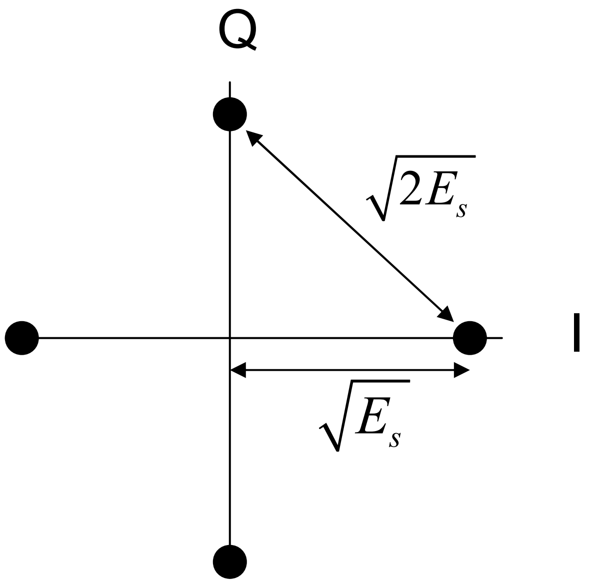
Quadrature Phase Shift Keying

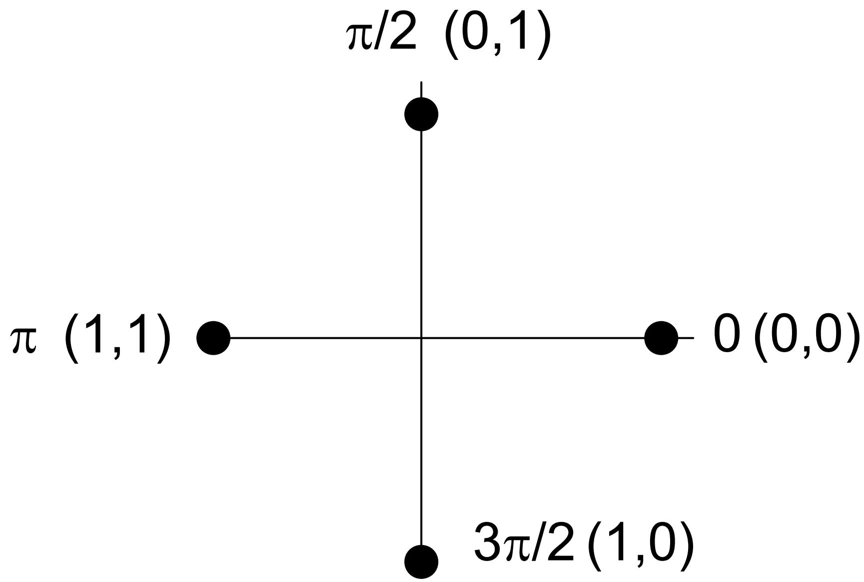
In quadrature phase shift keying (QPSK) the phase of the carrier takes on one of four equally spaced values such as 0, π/2, π, and 3π/2, where each value of the phase corresponds to a unique pair of bits.

The QPSK signal may be written as:

$$s(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right)$$

where i=1,2,3,4. Observe that two bits are transmitted in a single modulated symbol.





Quadrature Phase Shift Keying

From the constellation diagram of the QPSK signal it can be seen that the distance between adjacent point is $\sqrt{2E_s}$. *Since each symbol corresponds to two bits, then $E_s=2E_b$, thus the distance between two neighboring points in the QPSK constellation is equal to $2\sqrt{E_b}$. Substituting this into the general definition of error probability:*

$$P_e \leq \sum_{\substack{j=1 \\ j \neq i}} Q\left(\frac{d_{i,j}}{\sqrt{2N_0}}\right)$$

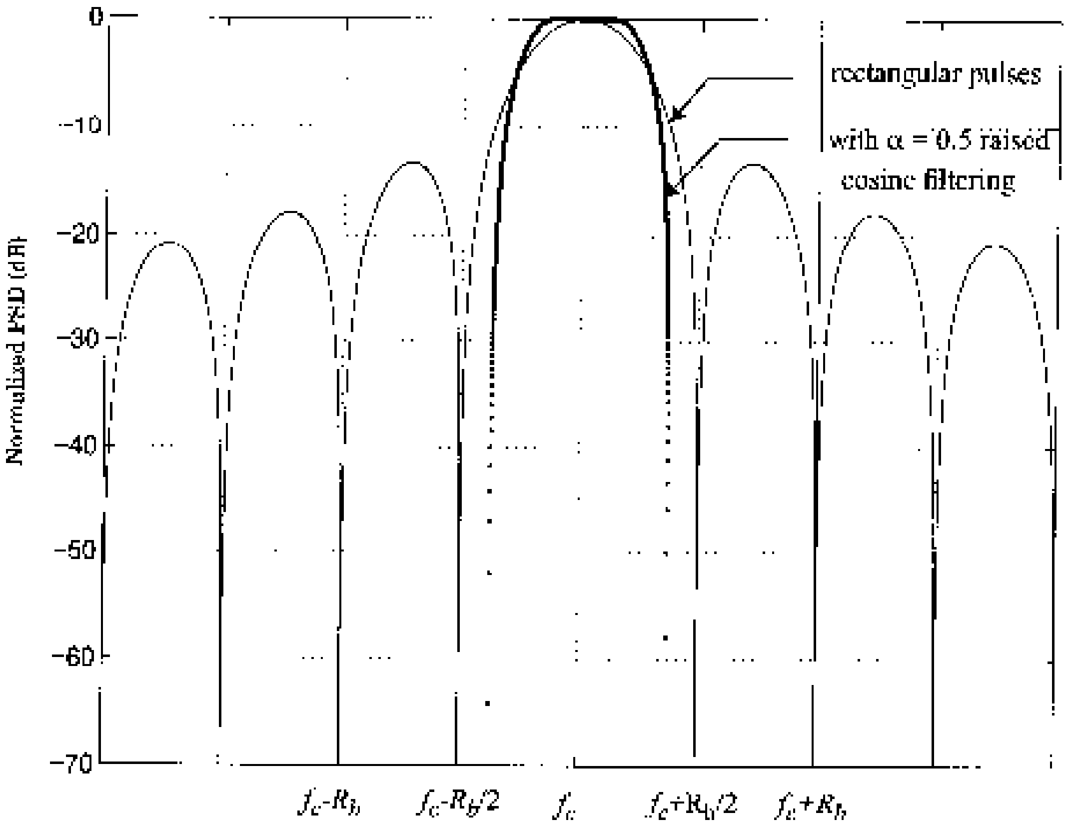
where $d_{i,j}$ is Euclidean distance between i^{th} and j^{th} signal point in the constellation gives an error probability for QSPK according to:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Quadrature Phase Shift Keying

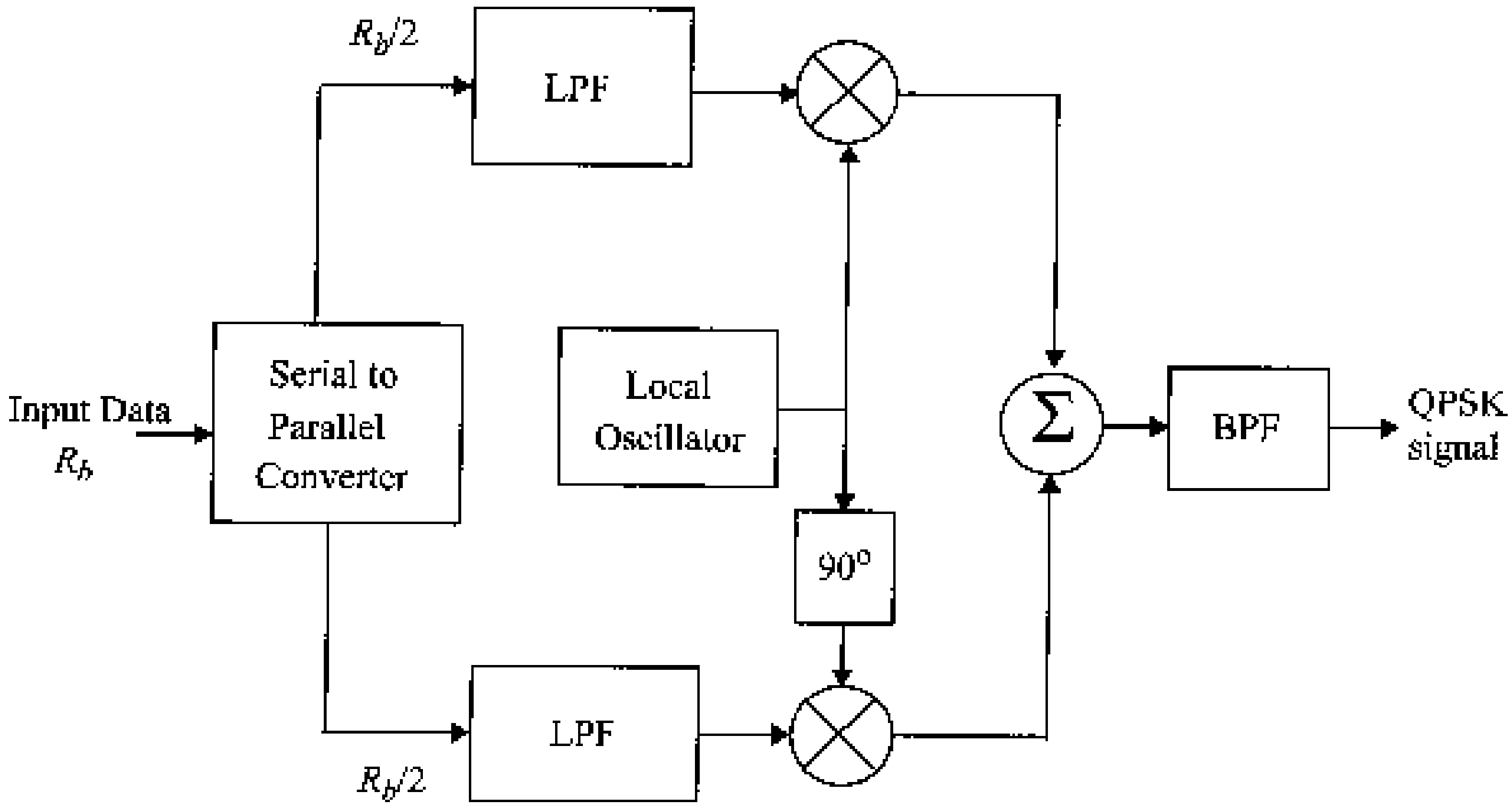
The power spectral density of the QSPK baseband signal can be expressed as:

$$\theta(f) = 4E_b \left(\frac{\sin(2\pi f T_s)}{(\pi f T_s)} \right)^2$$



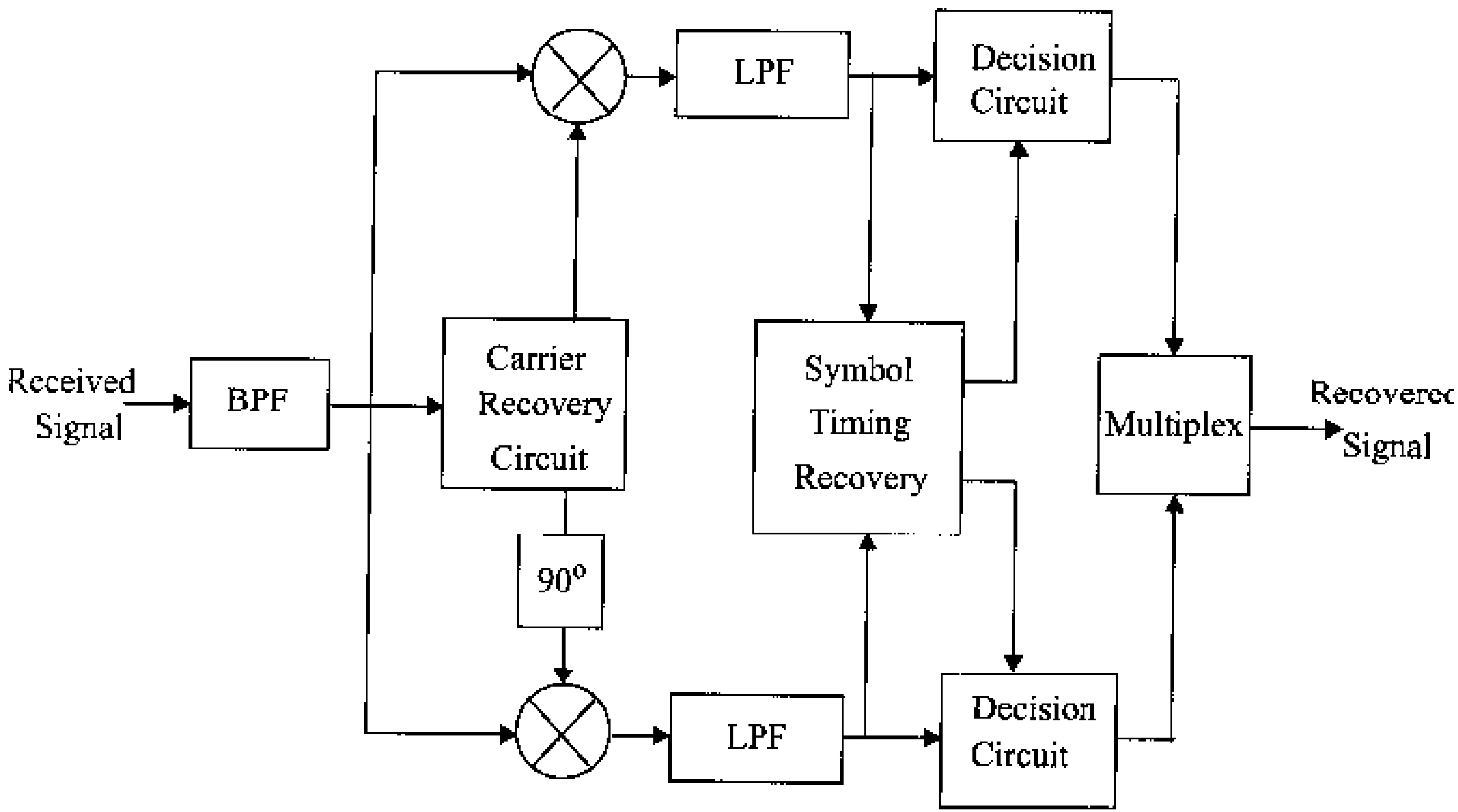
Quadrature Phase Shift Keying

QPSK transmitter



Quadrature Phase Shift Keying

QPSK receiver



M-ary Phase Shift Keying

In M-ary PSK, the carrier phase assigns one out of M possible values, namely, $\theta_i = 2(i-1)\pi / M$, where $i = 1, 2 \ldots M$.

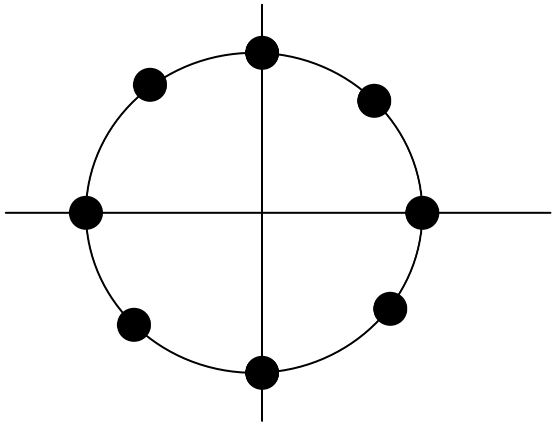
The number of phases are often chosen as $M=2^K$. This allows for the simultaneous transmission of $K=\log_2 M$ bits.

The modulated signal can be written as:

$$s(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right)$$

Where $E_s=E_bK$ is the energy per symbol, and $T_s=T_bK$ is the symbol period.

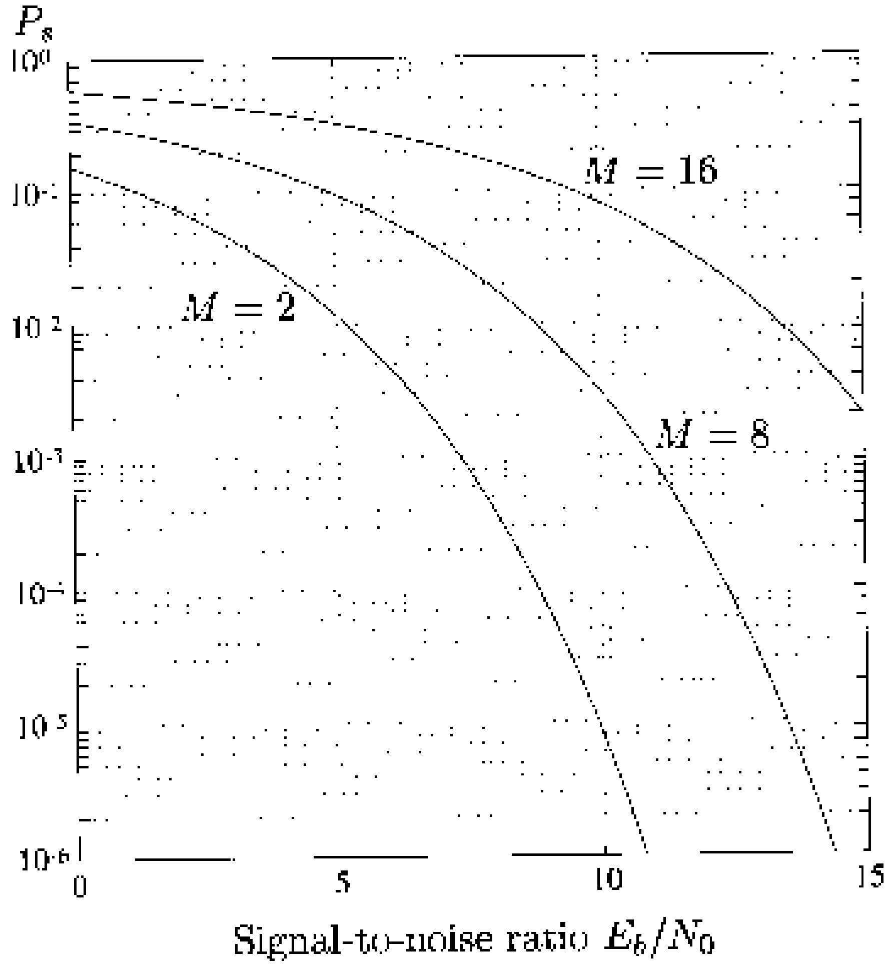
M-ary Phase Shift Keying



The signals points are equally spaced on a circle with radius $\sqrt{E_s}$. From the geometry it is easy seen that the distance between adjacent symbols is equal to $2\sqrt{E_s}\sin(\pi/M)$. Hence, the average symbol error of an M-ary PSK system is upper bound by:

$$P_s \le 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

M-ary Phase Shift Keying



M-ary Phase Shift Keying

The power spectral density of the M-ary PSK baseband signal can be expressed as:

$$\theta(f) = 2E_b \log_2 M \left(\frac{\sin(2\pi f \log_2 M)}{2\pi f \log_2 M} \right)^2$$