

Ex 4) LGS Differenzgleichungen

$$\begin{cases} Y_{n+2} - 5Y_{n+1} + 6Y_n = 2n+1 \\ Y_0 = Y_1 = 1 \end{cases}$$

1)  $Y_{hn}$ : Ansatz:  $r^2 - 5r + 6 = 0$   
 $\Leftrightarrow r_1 = 3, r_2 = 2$   
 $\therefore \underline{\underline{Y_{hn} = C_1 \cdot 3^n + C_2 \cdot 2^n}}$

2)  $Y_{pn}$ : Ansatz:  $Y_{pn} = an + b$   
Ins. in LGS:

$$\begin{aligned} & Y_{n+2} - 5Y_{n+1} + 6Y_n \\ &= a(n+2) + b - 5(a(n+1) + b) + 6(an + b) \\ &= n(a - 5a + 6a) + 2a + b - 5a - 5b + 6b \\ &= 2an - 3a + 2b = 2n + 1 \end{aligned}$$

$$\Leftrightarrow \begin{cases} 2a = 2 \\ -3a + 2b = 1 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = 2 \end{cases}$$

$\therefore \underline{\underline{Y_{pn} = n + 2}}$

3) Allm. Lsg:

$$\underline{\underline{Y_n = Y_{hn} + Y_{pn} = C_1 \cdot 3^n + C_2 \cdot 2^n + n + 2}}$$

$$Y_0 = C_1 \cdot 3^0 + C_2 \cdot 2^0 + 0 + 2 = C_1 + C_2 + 2 = 1 \Leftrightarrow C_1 = -C_2 - 1$$

$$\begin{aligned} Y_1 &= C_1 \cdot 3^1 + C_2 \cdot 2^1 + 1 + 2 = 3C_1 + 2C_2 + 3 \\ &= 3(-C_2 - 1) + 2C_2 + 3 = -C_2 = 1 \end{aligned}$$

$$\Leftrightarrow C_1 = 0 \quad C_2 = -1$$

Subst. Lsg:  $\underline{\underline{Y_n = n + 2 - 2^n}}$

Ex 5) LGS Differenzgleichungen

$$\begin{cases} Y_{n+2} - 4Y_{n+1} + 3Y_n = 4n + 4 \\ Y_0 = 1, Y_1 = 0 \end{cases}$$

1) Y<sub>hom</sub>: charakter. eq:  $r^2 - 4r + 3 = 0$   
 $\Leftrightarrow r_1 = 1 \quad r_2 = 3$

$Y_{hom} = C_1 \cdot 1^n + C_2 \cdot 3^n = C_1 + C_2 \cdot 3^n$

2) Y<sub>pn</sub>: Standardansatz:  $Y_{pn} = an + b$  ← Fehler!  
 $\Rightarrow Y_{pn} = n(an + b) = an^2 + bn$  ← Finns mte  
i Y<sub>hom</sub>. Ok!

Ins:  $Y_{n+2} - 4Y_{n+1} + 3Y_n =$

$$= a(n+2)^2 + b(n+2) - 4(a(n+1)^2 + b(n+1)) + 3(an^2 + bn)$$

$$= n^2(a - 4a + 3a) + n(4a + b - 8a - 4b + 3b)$$

$$+ 4a + 2b - 4a - 4b$$

$$= 0n^2 - 4an - 2b = 4n + 4$$

$$\Leftrightarrow \begin{cases} -4a = 4 \\ -2b = 4 \end{cases} \Leftrightarrow \begin{cases} a = -1 \\ b = -2 \end{cases}$$

$\Rightarrow Y_{pn} = -n^2 - 2n$

3) Allm. Lsg:

$Y_n = Y_{hom} + Y_{pn} = C_1 + C_2 \cdot 3^n - n^2 - 2n$

$$Y_0 = C_1 + C_2 \cdot 3^0 - 0 - 0 = C_1 + C_2 = 1 \Leftrightarrow C_1 = 1 - C_2$$

$$Y_1 = C_1 + C_2 \cdot 3^1 - 1^2 - 2 \cdot 1 = C_1 + 3C_2 - 3 = 1 - C_2 + 3C_2 - 3 = 2C_2 - 2 = 0 \Leftrightarrow C_1 = 0 \quad C_2 = 1$$

Subst. Lsg:

$Y_n = 3^n - n^2 - 2n$