Ex 5 Bestäm

a) De^x b) $D\ln x$ c) $D\sin x$

a) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{e^{x+h}-e^x}{h} = e^x\dfrac{(e^h-1)}{h} \to e^x \cdot 1 = e^x$ då $h \to 0$ ($\dfrac{e^h-1}{h} \to 1$)

$\therefore \boxed{De^x = e^x}$

b) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\ln(x+h)-\ln x}{h} = \dfrac{\ln\left(\frac{x+h}{x}\right)}{h} = \dfrac{\ln\left(1+\frac{h}{x}\right)}{\frac{h}{x}} \cdot \dfrac{1}{x}$ ($\dfrac{\ln(1+\frac{h}{x})}{\frac{h}{x}} \to 1$)

$\to 1 \cdot \dfrac{1}{x} = \dfrac{1}{x}$ då $h \to 0$

$\therefore \boxed{D\ln x = \dfrac{1}{x}}$

c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sin(x+h)-\sin x}{h} = \dfrac{\sin x\cos h + \sin h\cos x - \sin x}{h}$

$= \sin x \cdot \dfrac{\cos h - 1}{h} + \cos x \cdot \dfrac{\sin h}{h}$ ($\dfrac{\sin h}{h} \to 1$, $\dfrac{\cos h - 1}{h} \to$?)

c) (forts)

$$\frac{\cos h - 1}{h} = \frac{(\cos h - 1)(\cos h + 1)}{h(\cos h + 1)} = \frac{\cos^2 h - 1}{h(\cos h + 1)} = -\frac{1 - \cos^2 h}{h(\cos h + 1)}$$

$$= -\frac{\sin^2 h}{h(\cos h + 1)} = -\frac{\sin h}{h} \cdot \frac{\sin h}{h} \cdot h \cdot \frac{1}{\cos h + 1} \to 1 \cdot 1 \cdot 0 \cdot \frac{1}{1+1} = 0$$

då $h \to 0$

$\therefore$ $\boxed{D \sin x = \cos x}$

Ex 9 Bestäm

a) $D\cos x$ b) $D x^\alpha$ c) $D a^x$ d) $D\frac{1}{x}$

a) $D\cos x = D\sin\left(\frac{\pi}{2} - x\right) = \cos\left(\frac{\pi}{2} - x\right)\cdot(-1) = -\sin x$

(kedjeregeln)

$\therefore$ $\boxed{D\cos x = -\sin x}$

b) $x > 0$: $D x^\alpha = D e^{\ln x^\alpha} = D e^{\alpha \ln x} = e^{\alpha \ln x}\cdot\frac{\alpha}{x} = x^\alpha\cdot\frac{\alpha}{x} = \underline{\alpha x^{\alpha-1}}$

$x < 0$: $x < 0 \Leftrightarrow -x > 0$

$$D x^\alpha = D(-1)^\alpha(-x)^\alpha = D(-1)^\alpha e^{\ln(-x)^\alpha} = D(-1)^\alpha e^{\alpha\ln(-x)}$$

kedje-regeln $\rightarrow$ $= (-1)^\alpha e^{\alpha\ln(-x)}\cdot\frac{\alpha}{-x}\cdot(-1) = (-1)^\alpha\cdot(-x)^\alpha\cdot\frac{\alpha}{-x}\cdot(-1)$

$$= (-1)^\alpha(-x)^\alpha\cdot\frac{\alpha}{x} = x^\alpha\cdot\frac{\alpha}{x} = \underline{\alpha x^{\alpha-1}}$$

$x = 0$: $(\Rightarrow \alpha > 0)$

$$\frac{f(0+h) - f(0)}{h} = \frac{(0+h)^\alpha - 0^\alpha}{h} = \frac{h^\alpha}{h} = h^{\alpha-1} \rightarrow \underline{0} \text{ då } h \rightarrow 0$$

$\therefore$ $\boxed{D x^\alpha = \alpha x^{\alpha-1}}$

c) $D\,a^x = D\,e^{\ln a^x} = D\,e^{x \ln a} = e^{x \ln a} \cdot \ln a = a^x \ln a$

Kedjeregeln

$\therefore \boxed{D\,a^x = a^x \ln a}$

d) $D\,\frac{1}{x} = D\,x^{-1} = (-1) \cdot x^{-2} = -\frac{1}{x^2}$

$\therefore \boxed{D\,\frac{1}{x} = -\frac{1}{x^2}}$