Ex 12 Lös

$$\begin{cases} y'' - 3y' + 2y = 2xe^{x} \\ y(0) = 1,\ y'(0) = 0 \end{cases}$$

1. y_h:

Karakt. ekv: $r^2 - 3r + 2 = 0 \Leftrightarrow r_1 = 1,\ r_2 = 2$

$\Rightarrow \quad y_h(x) = C_1 e^{x} + C_2 e^{2x}$

2. y_p:

Ansats: $y_p = ze^{x} \Rightarrow y_p' = z'e^{x} + ze^{x} = (z' + z)e^{x}$

$\Rightarrow y_p'' = (z'' + z')e^{x} + (z' + z)e^{x} = (z'' + 2z' + z)e^{x}$

Ins i ekv:

$y'' - 3y' + 2y = (z'' + 2z' + z - 3(z' + z) + 2z)e^{x} = (z'' - z')e^{x} = 2xe^{x}$

$\Leftrightarrow \quad z'' - z' = 2x$

Ansats: $z_p = (ax + b)x = ax^2 + bx \Rightarrow z_p' = 2ax + b \Rightarrow z_p'' = 2a$

Ins i ekv:

$z'' - z' = 2a - (2ax + b) = \underbrace{-2a}_{=2}x + \underbrace{2a - b}_{=0} = 2x \quad \Leftrightarrow \quad a = -1,\ b = -2$

Ex 12

$$\therefore \quad y_p(x) = -(x^2+2x)e^x$$

Allm. lsg: $y(x) = y_h(x) + y_p(x) = C_1 e^x + C_2 e^{2x} - (x^2+2x)e^x$

$$y(0) = C_1 e^0 + C_2 e^0 - (0^2+2\cdot 0)e^0 = C_1 + C_2 = 1 \Leftrightarrow C_1 = 1 - C_2$$

$$y'(x) = C_1 e^x + C_2 e^{2x}\cdot 2 - (2x+2)e^x - (x^2+2x)e^x$$

$$\Rightarrow y'(0) = C_1 e^0 + 2C_2 e^0 - (2\cdot 0+2)e^0 - (0^2+2\cdot 0)e^0 = C_1 + 2C_2 - 2$$

$$= 1 - C_2 + 2C_2 - 2 = C_2 - 1 = 0 \Leftrightarrow C_1 = 0, \; C_2 = 1$$

$\therefore$ Slut lsg: $y(x) = e^{2x} - (x^2+2x)e^x$.